Calculations relevant to ELT MEIFU Study

Simon Morris 9.9.02

$\mathrm{Angstrom} := 10^{-10}\cdot \mathrm{m}$ $\qquad \mu\mathrm{m} := 10^{-6}\cdot \mathrm{m}$ $\qquad \mathrm{nm} := 10^{-9}\cdot \mathrm{m}$

$\mathrm{Planck} := 6.626076\cdot 10^{-34}\cdot \mathrm{J}\cdot \mathrm{s}$ $\qquad \lambda_{\mathrm{Ly}\alpha} := 1215.67\cdot \mathrm{Angstrom}$

$c := 2.9979270\cdot 10^{10}\cdot \frac{\mathrm{cm}}{\mathrm{sec}}$ $\qquad \mathrm{Photon} := 1$ $\qquad \mathrm{arcsec} := \frac{\mathrm{deg}}{60\cdot 60}$ $\qquad \mathrm{arcmin} := \frac{\mathrm{deg}}{60.0}$

$\mathrm{pc} := 3.085678\cdot 10^{18}\cdot \mathrm{cm}$ $\qquad \mathrm{kpc} := \mathrm{pc}\cdot 10^{3}$ $\qquad \mathrm{Mpc} := \mathrm{pc}\cdot 10^{6}$ $\qquad \mathrm{Gyr} := 10^{9}\cdot \mathrm{yr}$

$\mathrm{nJy} := 10^{-9}\cdot 10^{-23}\cdot \mathrm{erg}\cdot \mathrm{cm}^{-2}\cdot \mathrm{s}^{-1}\cdot \mathrm{Hz}^{-1}$ $\qquad \mathrm{AB}(F) := -2.5\cdot \log\left[\frac{F}{\left(\mathrm{erg}\cdot \mathrm{cm}^{-2}\cdot \mathrm{sec}^{-1}\cdot \mathrm{Hz}^{-1}\right)}\right] - 48.60$

Cosmology from Carroll, Press and Turner, 1992, ARAA, 30, 499

$H_0 := h\cdot 100\cdot \frac{\mathrm{km}}{\mathrm{sec}\cdot \mathrm{Mpc}}$ $\qquad H_0^{-1} = 1.397\times 10^{10}\,\mathrm{yr}$

$\Omega_M \equiv 0.3$ $\qquad h \equiv 0.7$ $\qquad \Omega_\Lambda \equiv 0.7$

$\Omega_k := 1 - \Omega_M - \Omega_\Lambda$ $\qquad \Omega_k = 0$

$q_0 := \frac{1}{2}\cdot \Omega_M - \Omega_\Lambda$ $\qquad q_0 = -0.55$

Now calculate proper distance

$$d_M(z) := \begin{cases} \frac{c}{H_0}\cdot \frac{1}{\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}}\cdot \sinh\left[\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}\cdot \int_0^z \left[(1+z)^2\cdot\left(1+\Omega_M\cdot z\right) - z\cdot(2+z)\cdot\Omega_\Lambda\right]^{-\frac{1}{2}} dz\right] & \text{if } \Omega_k > 0 \\ \frac{c}{H_0}\cdot \frac{1}{\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}}\cdot \sin\left[\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}\cdot \int_0^z \left[(1+z)^2\cdot\left(1+\Omega_M\cdot z\right) - z\cdot(2+z)\cdot\Omega_\Lambda\right]^{-\frac{1}{2}} dz\right] & \text{if } \Omega_k < 0 \\ \frac{c}{H_0}\cdot \int_0^z \left[(1+z)^2\cdot\left(1+\Omega_M\cdot z\right) - z\cdot(2+z)\cdot\Omega_\Lambda\right]^{-\frac{1}{2}} dz & \text{otherwise} \end{cases}$$

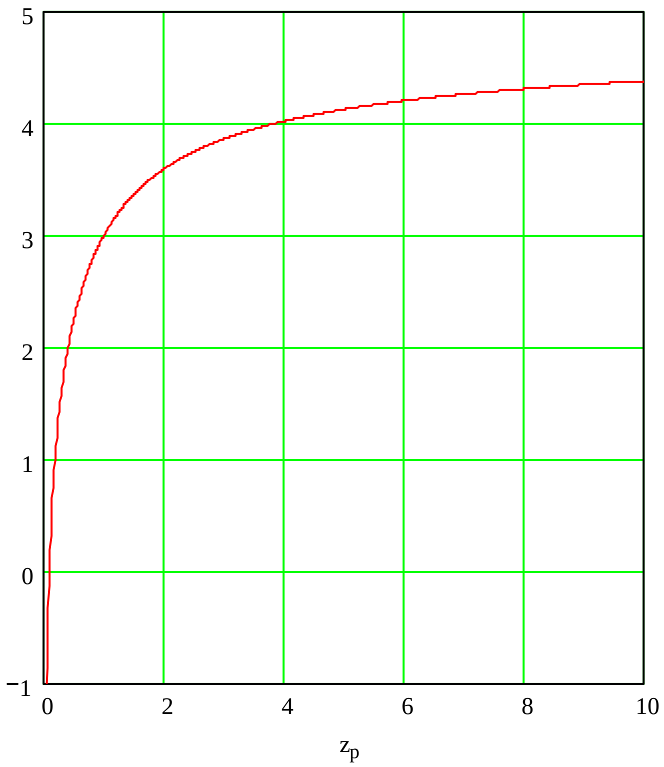
$d_L(z) := (1 + z)\cdot d_M(z)$ Luminosity distance $d_A(z) := \frac{1}{1+z}\cdot d_M(z)$ Angular diameter distance

Now do Volume out to some redshift

$$V(d_M) := \begin{cases} \left(\frac{c}{H_0}\right)^3\cdot\frac{4\cdot\pi}{2\cdot\Omega_k}\cdot\left[\left[\frac{d_M}{c}\cdot H_0\cdot\left[1+\Omega_k\cdot\left(\frac{H_0}{c}\cdot d_M\right)^2\right]\right]^{\frac{1}{2}} - \frac{1}{\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}}\cdot \operatorname{asinh}\left[\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}\cdot\frac{H_0}{c}\cdot d_M\right]\right] & \text{if } \Omega_k \\ \left(\frac{c}{H_0}\right)^3\cdot\frac{4\cdot\pi}{2\cdot\Omega_k}\cdot\left[\left[\frac{d_M}{c}\cdot H_0\cdot\left[1+\Omega_k\cdot\left(\frac{H_0}{c}\cdot d_M\right)^2\right]\right]^{\frac{1}{2}} - \frac{1}{\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}}\cdot \operatorname{asin}\left[\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}\cdot\frac{H_0}{c}\cdot d_M\right]\right] & \text{if } \Omega_k \cdot \\ \frac{4\cdot\pi}{3}\cdot d_M{}^3 & \text{otherwise} \end{cases}$$

Volume per square degree out to given redshift

$$\log\left(\frac{V(d_M(z_p))}{\frac{4\cdot\pi}{\text{arcmin}^2}\cdot \text{Mpc}^3}\right)$$



Volume between z=5 and 6.5 per square arcmin

$$\frac{V\left(d_M(6.5)\right) - V\left(d_M(5)\right)}{\frac{4\cdot\pi}{\mathrm{arcmin}^2}} = 3.739\times 10^3\,\mathrm{Mpc}^3$$

Check on redshift ranges and resolutions for default 3 band MEIFU design

$R := 1750$

$\lambda_{1b} := 6000\cdot\mathrm{Angstrom}$ $\quad \lambda_{1r} := 7110\cdot\mathrm{Angstrom}$ $\quad \lambda_{1c} := \frac{\lambda_{1b} + \lambda_{1r}}{2}$ $\quad \lambda_{1c} = 6.555\times 10^3\,\mathrm{Angstrom}$

$z_{1b} := \frac{\lambda_{1b} - \lambda_{Ly\alpha}}{\lambda_{Ly\alpha}}$ $\quad z_{1b} = 3.936$

$\Delta\lambda_1 := \frac{\lambda_{1c}}{2\cdot R}$ $\quad \Delta\lambda_1 = 1.873\,\mathrm{Angstrom}$ dispersion per pixel

$\mathrm{npix}_{\lambda 1} := \frac{\lambda_{1r} - \lambda_{1b}}{\Delta\lambda_1}$ $\quad \mathrm{npix}_{\lambda 1} = 592.677$ number of pixels in wavelength direction

$\lambda_{2b} := 7110\cdot\mathrm{Angstrom}$ $\quad \lambda_{2r} := 8430\cdot\mathrm{Angstrom}$ $\quad \lambda_{2c} := \frac{\lambda_{2b} + \lambda_{2r}}{2}$ $\quad \lambda_{2c} = 7.77\times 10^3\,\mathrm{Angstrom}$

$z_{2b} := \frac{\lambda_{2b} - \lambda_{Ly\alpha}}{\lambda_{Ly\alpha}}$ $\quad z_{2b} = 4.849$

$\Delta\lambda_2 := \frac{\lambda_{2c}}{2\cdot R}$ $\quad \Delta\lambda_2 = 2.22\,\mathrm{Angstrom}$ dispersion per pixel

$\mathrm{npix}_{\lambda 2} := \frac{\lambda_{2r} - \lambda_{2b}}{\Delta\lambda_2}$ $\quad \mathrm{npix}_{\lambda 2} = 594.595$ number of pixels in wavelength direction

$\lambda_{3b} := 8430\cdot\mathrm{Angstrom}$ $\quad \lambda_{3r} := 10000\cdot\mathrm{Angstrom}$ $\quad \lambda_{3c} := \frac{\lambda_{3b} + \lambda_{3r}}{2}$ $\quad \lambda_{3c} = 9.215\times 10^3\,\mathrm{Angstrom}$

$z_{3b} := \frac{\lambda_{3b} - \lambda_{Ly\alpha}}{\lambda_{Ly\alpha}}$ $\quad z_{3b} = 5.934$ $\quad z_{3r} := \frac{\lambda_{3r} - \lambda_{Ly\alpha}}{\lambda_{Ly\alpha}}$ $\quad z_{3r} = 7.226$

$\Delta\lambda_3 := \frac{\lambda_{3c}}{2\cdot R}$ $\quad \Delta\lambda_3 = 2.633\,\mathrm{Angstrom}$ dispersion per pixel

$\mathrm{npix}_{\lambda 3} := \frac{\lambda_{3r} - \lambda_{3b}}{\Delta\lambda_3}$ $\quad \mathrm{npix}_{\lambda 3} = 596.31$ number of pixels in wavelength direction

$\Delta z_1 := z_{2b} - z_{1b}$ $\Delta z_1 = 0.913$

$\Delta z_2 := z_{3b} - z_{2b}$ $\Delta z_2 = 1.086$

$\Delta z_3 := z_{3r} - z_{3b}$ $\Delta z_3 = 1.291$

Calculate some volumes in various surveys

$$V_{survey}(arcminx, arcminy, z1, z2) := \frac{\frac{V\left(d_M(z2)\right) - V\left(d_M(z1)\right)}{4\cdot\pi}}{arcminx\cdot arcminy\,(arcmin)^2}$$

$V_{survey}\left(2.68, 3.01, z_{1b}, z_{3r}\right) = 6.772\times 10^4\,Mpc^3$ MEIFU

$V_{survey}(0.68, 0.55, 2.95, 4.44) = 1.727\times 10^3\,Mpc^3$ SAURON 2.5e-18 in 20 hours on 4m

$V_{survey}(6, 6, 3.40, 3.46) = 6.869\times 10^3\,Mpc^3$ Fynbo typical table entry, 2e-17

$V_{survey}(36, 36, 4.37, 4.57) = 7.438\times 10^5\,Mpc^3$ Rhoads et al., 2e-17 in 6 hours on 4m

Geometry Calculations for MEIFU concept

Calculate number of slitlets

$pix := 0.1\cdot arcsec$ $FOV_{tot} := 2.84\cdot 60\cdot arcsec$

$slit_L := 1.2\cdot arcsec$ $Nslit_L := \frac{FOV_{tot}}{slit_L}$ $Nslit_L = 142$

$slit_W := 0.2\cdot arcsec$ $Nslit_W := \frac{FOV_{tot}}{slit_W}$ $Nslit_W = 852$

$Nslit_{tot} := Nslit_L\cdot Nslit_W$ $Nslit_{tot} = 1.21\times 10^5$

consider single 8k x 8k camera - calculate pixels used

$npix := 8\cdot 1024$ all calculations in pixels

Use Content numbers for 8m MEIFU geometry - i.e. stay with same slit size in pixels and same offsets for slits relative to each other.

$dx := 75$ $dy := 133.2$ offset between micoslits

$nx := \frac{npix}{dx}$ $ny := \frac{npix}{dy}$ $nx = 109.227$ $ny = 61.502$

$nslit := nx \cdot ny$ $nslit = 6.718 \times 10^3$

$slit\_len := 12$

total number of microslits on the detector

$nspec := nx \cdot ny \cdot slit\_len$ $nspec = 8.061 \times 10^4$

$spec\_len := 600$

$npix\_used := nspec \cdot spec\_len$ $npix\_used = 4.837 \times 10^7$

total number of pixels with data on them

$frac\_used := \frac{npix\_used}{npix \cdot npix}$ $frac\_used = 0.721$

now consider what that means in terms of sky coverage

consider 12 x 2 pixel microslits, 0.1 arcsec per pixel in both directions

$dx\_slit\_arcsec := slit\_len \cdot 0.1 \cdot arcsec$ $dy\_slit\_arcsec := 2 \cdot 0.1 \cdot arcsec$

$area := nslit \cdot dx\_slit\_arcsec \cdot dy\_slit\_arcsec$

$area = 0.448\, arcmin^2$

area on sky covered by 1 spectrograph

for 24 spectrographs get:

$tot\_area := 24 \cdot area$ $tot\_area = 10.748\, arcmin^2$

equivalent to:

$\sqrt{tot\_area} = 3.278\, arcmin$

$npix \cdot npix \cdot 24 = 1.611 \times 10^9$ total number of detector pixels in all spectrographs

Do some S/N calculations for fiducial redshift

$z := 6$ $\lambda_{Ly\alpha} \cdot (1 + z) = 0.851\,\mu m$

Consider and 30m telescope with a total throughput of 27%

$r := 15 \cdot m$ $\varepsilon := 0.27$ $expose := 4 \cdot 8 \cdot 3600 \cdot s$ $expose = 1.152 \times 10^{5}\,s$

Night Sky values for Optical/IR

Magnitude central wavelengths and zero points from ESO web site
http://www.eso.org/observing/etc/doc/gen/formulaBook/node12.html

$$\lambda_0 := \begin{pmatrix} 0.360 \\ 0.440 \\ 0.550 \\ 0.640 \\ 0.790 \\ 0.950 \\ 1.250 \\ 1.650 \\ 2.200 \\ 3.500 \\ 4.800 \end{pmatrix} \cdot \mu m \qquad Z_{BAND} := \begin{pmatrix} 7.3788 \\ 7.1804 \\ 7.4425 \\ 7.6408 \\ 7.9115 \\ 8.1101 \\ 8.4989 \\ 8.9706 \\ 9.4367 \\ 10.2649 \\ 10.2692 \end{pmatrix}$$

ESO Night Sky Brightnesses at new moon (magnitudes per square arcsec)
(IR magnitudes are actually Ks, L and M-NB)

$$S := \begin{pmatrix} 22.0 \\ 22.7 \\ 21.8 \\ 20.9 \\ 19.9 \\ 18.8 \\ 16.5 \\ 14.4 \\ 13.0 \\ 3.9 \\ 1.2 \end{pmatrix}$$

Calculate flux per square arcsec

$$F_S := \overrightarrow{10^{-\frac{S}{2.5}-Z_{BAND}}} \cdot W \cdot m^{-2} \cdot \mu m^{-1} \cdot arcsec^{-2}$$

$F_S =$

	0
0	$6.625 \cdot 10^{-17}$
1	$5.49 \cdot 10^{-17}$
2	$6.879 \cdot 10^{-17}$
3	$9.982 \cdot 10^{-17}$
4	$1.344 \cdot 10^{-16}$
5	$2.344 \cdot 10^{-16}$
6	$7.963 \cdot 10^{-16}$
7	$1.86 \cdot 10^{-15}$
8	$2.308 \cdot 10^{-15}$
9	$1.497 \cdot 10^{-12}$
10	$1.782 \cdot 10^{-11}$

$W \cdot m^{-2} \cdot \mu m^{-1} \cdot arcsec^{-2}$

Calculate Photons per square arcsec

$$P_S := \overrightarrow{\left(\frac{F_S \cdot \lambda_0}{Planck \cdot c} Photon\right)}$$

$P_S =$

	0
0	120.067
1	121.612
2	190.452
3	321.59
4	534.625
5	$1.121 \cdot 10^{3}$
6	$5.011 \cdot 10^{3}$
7	$1.545 \cdot 10^{4}$
8	$2.556 \cdot 10^{4}$
9	$2.637 \cdot 10^{7}$
10	$4.305 \cdot 10^{8}$

$Photon \cdot s^{-1} \cdot m^{-2} \cdot \mu m^{-1} \cdot arcsec^{-2}$

Record Content 1996 equivalent background reduction factors to get effective continuum between OH lines:

J=15 H=38 K=4.4 - divide above J,H,K fluxes by this number for OH supressed background.

fitting function

$$Sky(\lambda) := linterp(\lambda_0, P_S, \lambda)$$

$$Sky(0.9 \cdot \mu m) = 937.652\, Photon \cdot s^{-1} \cdot m^{-2} \cdot \mu m^{-1} \cdot arcsec^{-2}$$

Flux to photon rate conversion

$F = 2.6 \times 10^{-19}\, erg \cdot cm^{-2} \cdot s^{-1}$ tune choice to get 5 sigma detection in 4 nights

$L := 4 \cdot \pi \cdot d_L(z)^2 \cdot F$ $L = 1.037 \times 10^{41}\, erg \cdot s^{-1}$ corresponding luminosity

$P := \frac{F \cdot \lambda_{Ly\alpha} \cdot (1 + z)}{Planck \cdot c}$ $P = 1.114 \times 10^{-7}\, cm^{-2} \cdot s^{-1}$

$N_{tot} := P \cdot \pi \cdot r^2 \cdot \varepsilon$ $N_{tot} = 0.213\, s^{-1}$ $S := N_{tot} \cdot expose$

$S = 2.449 \times 10^4$ total Lyalpha photons detected in an exposure

$apfac := 0.5$ assume sum over an aperture aperture which only gets 1/2 of the photons (actually will use optimal weighting and depens on seeing and object morphology)

Figure out sky background

assume object flux (and hence sky flux) is summed over various different sized regions

assume line flux is all within 4 pixels spectrally (=2xfwhm, crit samp)

'Natural Seeing'

$$N_{totsky} := Sky\left[\lambda_{Ly\alpha}\cdot(1+z)\right]\cdot 2\cdot\frac{\lambda_{Ly\alpha}\cdot(1+z)}{R}\cdot(0.6\cdot arcsec)^2\cdot\pi\cdot r^2\cdot\varepsilon \qquad N_{totsky} = 50.65\,s^{-1}$$

$B := N_{totsky}\cdot expose$ $\qquad B = 5.835\times 10^6$ total sky photons

$$SN := \frac{S\cdot apfac}{\sqrt{S\cdot apfac + B}} \qquad SN = 5.064$$ resulting S/N

AO with fuzzy targets

$$N_{totsky} := Sky\left[\lambda_{Ly\alpha}\cdot(1+z)\right]\cdot 2\cdot\frac{\lambda_{Ly\alpha}\cdot(1+z)}{R}\cdot(0.2\cdot arcsec)^2\cdot\pi\cdot r^2\cdot\varepsilon \qquad N_{totsky} = 5.628\,s^{-1}$$

$B := N_{totsky}\cdot expose$ $\qquad B = 6.483\times 10^5$ total sky photons

$$SN := \frac{S\cdot apfac}{\sqrt{S\cdot apfac + B}} \qquad SN = 15.065$$ resulting S/N

Diffraction Limited, 0.79 microns, 30m (i.e. point source, perfect AO - fantasy)

$D_{EE50\%} := 5.9\cdot 10^{-3}\cdot arcsec$ haven't bothered to copy over real calculation for this, and so is hardwired - obviously varies with wavelength in practice

$$N_{totsky} := Sky\left[\lambda_{Ly\alpha}\cdot(1+z)\right]\cdot 2\cdot\frac{\lambda_{Ly\alpha}\cdot(1+z)}{R}\cdot\left(D_{EE50\%}\right)^2\cdot\pi\cdot r^2\cdot\varepsilon \qquad N_{totsky} = 4.898\times 10^{-3}\,s^{-1}$$

$B := N_{totsky}\cdot expose$ $\qquad B = 564.199$ total sky photons

$$SN := \frac{S\cdot apfac}{\sqrt{S\cdot apfac + B}} \qquad SN = 108.189$$ resulting S/N

$F \equiv 2.6\cdot 10^{-19}\cdot erg\cdot cm^{-2}\cdot s^{-1}$ Ly A eline flux transcribed here

Now try to make a ballpark comparison with an FTS (know in advance S/N will be a lot worse for strongly background limited, although are some subtleties about smoothing out the OH line noise)

Consider a scanning strategy where one observes for 3 x 4 nights (i.e. the time allocated to the 3 bands above), covering the same total wavelength range and aiming for a similar spectral resolution in the middle of the band (FTS reduced cube will be equally spaced in frequency, not in wavelength)

$M_{spec} := R \cdot 2$ — Spectral bins

$n_{samp} := M_{spec} \cdot 2$ — number of FTS samples (two sided)

$n_{samp} = 7 \times 10^3$

Have ignored stuff about powers of 2 above and requirements on sampling strategy, just going to order of magnitude estimates

$$expose_{FTS} := \frac{expose \cdot 3}{n_{samp}}$$ time per FTS step $expose_{FTS} = 49.371\,s$

$$\varepsilon_{FTS} := \frac{\varepsilon}{0.6}$$ allow the FTS to have higher throughput (no grism) - assuming 2 port setup. Ignoring readout noise, as did for IFU.

Consider 'broad band' S/N per step

$$Nsky_{BB} := \varepsilon_{FTS} \cdot expose_{FTS} \cdot 4 \cdot \pi \cdot r^2 \cdot \int_{\lambda_{1b}}^{\lambda_{3r}} Sky(\lambda)\, d\lambda$$

$Nsky_{BB} = 1.763 \times 10^7\ Photon \cdot arcsec^{-2}$ total number of photons detected from the sky for the entire wavelength range per step

FTS broad band S/N for pure emission line source is

$$P := \frac{F \cdot \lambda_{Ly\alpha} \cdot (1+z)}{Planck \cdot c}$$ $P = 1.114 \times 10^{-7}\, cm^{-2} \cdot s^{-1}$

$N_{tot} := P \cdot \pi \cdot r^2 \cdot \varepsilon_{FTS}$ $N_{tot} = 0.354\, s^{-1}$ $S := N_{tot} \cdot expose_{FTS}$

'Natural Seeing'

$B := Nsky_{BB} \cdot (0.6 \cdot arcsec)^2$ $B = 6.346 \times 10^6$ total sky photons

$$SN := \frac{S \cdot apfac}{\sqrt{S \cdot apfac + B}}$$ $SN = 3.472 \times 10^{-3}$ resulting S/N

Calculate the SNR in the Wavelength domain - equation from Graham et al 1998 (astro-ph/9803163), makes some sense - scales by sqrt of number of exposures, inversely with resolution. Additional factor to account for fact that is pure eline source. Factor is signal in wavelength range containing the eline, compared to the signal over the whole band (see Davis, Abrams and Brault 1999, Draft of book, for justification)

$$SN_{FTS} := \frac{\sqrt{n_{samp}} \cdot SN}{M_{spec}} \cdot \frac{\lambda_{3r} - \lambda_{1b}}{\left[2 \cdot \frac{\lambda_{Ly\alpha} \cdot (1 + z)}{R}\right]}$$ $SN_{FTS} = 0.034$

As expected the S/N sucks - to make FTS competitive would need to tune down to much narrower band. See main report for fuller discussion of when an FTS would be an attractive option.

AO with fuzzy targets

$B := Nsky_{BB} \cdot (0.2 \cdot arcsec)^2$ $B = 7.051 \times 10^5$ total sky photons

$$SN := \frac{S \cdot apfac}{\sqrt{S \cdot apfac + B}}$$ $SN = 0.01$ resulting S/N

$$SN_{FTS} := \frac{\sqrt{n_{samp}} \cdot SN}{M_{spec}} \cdot \frac{\lambda_{3r} - \lambda_{1b}}{\left[2 \cdot \frac{\lambda_{Ly\alpha} \cdot (1 + z)}{R}\right]}$$ $SN_{FTS} = 0.102$

Diffraction limited

$B := Nsky_{BB} \cdot \left(D_{EE50\%}\right)^2$ $B = 613.59$ total sky photons

$$SN := \frac{S \cdot apfac}{\sqrt{S \cdot apfac + B}}$$ $SN = 0.351$ resulting S/N

$$SN_{FTS} := \frac{\sqrt{n_{samp}} \cdot SN}{M_{spec}} \cdot \frac{\lambda_{3r} - \lambda_{1b}}{\left[2 \cdot \frac{\lambda_{Ly\alpha} \cdot (1 + z)}{R}\right]}$$ $SN_{FTS} = 3.447$

$\varsigma > 0$

< 0