Calculations on Lyman alpha surface brightnesses (S. Morris 9.9.02)

$\text{Angstrom} := 10^{-10} \cdot m$ $\qquad \mu m := 10^{-6} \cdot m$ $\qquad nm := 10^{-9} \cdot m$

$h_p := 6.626076 \cdot 10^{-34} \cdot J \cdot s$ $\qquad \lambda_{Ly\alpha} := 1215.67 \cdot \text{Angstrom}$

$c := 2.9979270 \cdot 10^{10} \cdot \frac{cm}{sec}$ $\qquad arcsec := \frac{deg}{60 \cdot 60}$ $\qquad arcmin := \frac{deg}{60.0}$

$pc := 3.085678 \cdot 10^{18} \cdot cm$ $\qquad kpc := pc \cdot 10^3$ $\qquad Mpc := pc \cdot 10^6$ $\qquad Gyr := 10^9 \cdot yr$

$eV := \frac{2.187691 \cdot 10^{-18} \cdot J}{13.6056981}$

Cosmology from Carroll, Press and Turner, 1992, ARAA, 30, 499

$H_0 := h \cdot 100 \cdot \frac{km}{sec \cdot Mpc}$ $\qquad H_0^{-1} = 1.397 \times 10^{10}\, yr$

$\Omega_M \equiv 0.3$ $\qquad h \equiv 0.7$ $\qquad \Omega_\Lambda \equiv 0.7$

$\Omega_k := 1 - \Omega_M - \Omega_\Lambda$ $\qquad \Omega_k = 0$

$q_0 := \frac{1}{2} \cdot \Omega_M - \Omega_\Lambda$ $\qquad q_0 = -0.55$

Now calculate proper distance

$$d_M(z) := \begin{cases} \frac{c}{H_0} \cdot \frac{1}{\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}} \cdot \sinh\left[\left(\left|\Omega_k\right|\right)^{\frac{1}{2}} \cdot \int_0^z \left[(1+z)^2 \cdot \left(1 + \Omega_M \cdot z\right) - z \cdot (2+z) \cdot \Omega_\Lambda\right]^{-\frac{1}{2}} dz\right] & \text{if } \Omega_k > 0 \\ \frac{c}{H_0} \cdot \frac{1}{\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}} \cdot \sin\left[\left(\left|\Omega_k\right|\right)^{\frac{1}{2}} \cdot \int_0^z \left[(1+z)^2 \cdot \left(1 + \Omega_M \cdot z\right) - z \cdot (2+z) \cdot \Omega_\Lambda\right]^{-\frac{1}{2}} dz\right] & \text{if } \Omega_k < 0 \\ \frac{c}{H_0} \cdot \int_0^z \left[(1+z)^2 \cdot \left(1 + \Omega_M \cdot z\right) - z \cdot (2+z) \cdot \Omega_\Lambda\right]^{-\frac{1}{2}} dz & \text{otherwise} \end{cases}$$

$d_L(z) := (1+z) \cdot d_M(z)$ Luminosity distance $\qquad d_A(z) := \frac{1}{1+z} \cdot d_M(z)$ Angular diameter distance

Now do Volume out to some redshift

$$V(d_M) := \begin{cases} \left(\frac{c}{H_0}\right)^3 \cdot \frac{4\cdot\pi}{2\cdot\Omega_k} \cdot \left[\left[\frac{d_M}{c} \cdot H_0 \cdot \left[1 + \Omega_k \cdot \left(\frac{H_0}{c} \cdot d_M \right)^2 \right] \right]^{\frac{1}{2}} - \frac{1}{\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}} \cdot \operatorname{asinh}\left[\left(\left|\Omega_k\right|\right)^{\frac{1}{2}} \cdot \frac{H_0}{c} \cdot d_M \right] \right] & \text{if } \Omega_k \text{ :} \\ \left(\frac{c}{H_0}\right)^3 \cdot \frac{4\cdot\pi}{2\cdot\Omega_k} \cdot \left[\left[\frac{d_M}{c} \cdot H_0 \cdot \left[1 + \Omega_k \cdot \left(\frac{H_0}{c} \cdot d_M \right)^2 \right] \right]^{\frac{1}{2}} - \frac{1}{\left(\left|\Omega_k\right|\right)^{\frac{1}{2}}} \cdot \operatorname{asin}\left[\left(\left|\Omega_k\right|\right)^{\frac{1}{2}} \cdot \frac{H_0}{c} \cdot d_M \right] \right] & \text{if } \Omega_k < \\ \frac{4\cdot\pi}{3} \cdot d_M^{\,3} & \text{otherwise} \end{cases}$$

Volume per square degree out to given redshift

$$\frac{V(d_M(6.5)) - V(d_M(5))}{\frac{4\cdot\pi}{\text{arcmin}^2}} = 3.739 \times 10^3 \,\text{Mpc}^3$$

Volume between z=5 and 6.5 per square arcmin

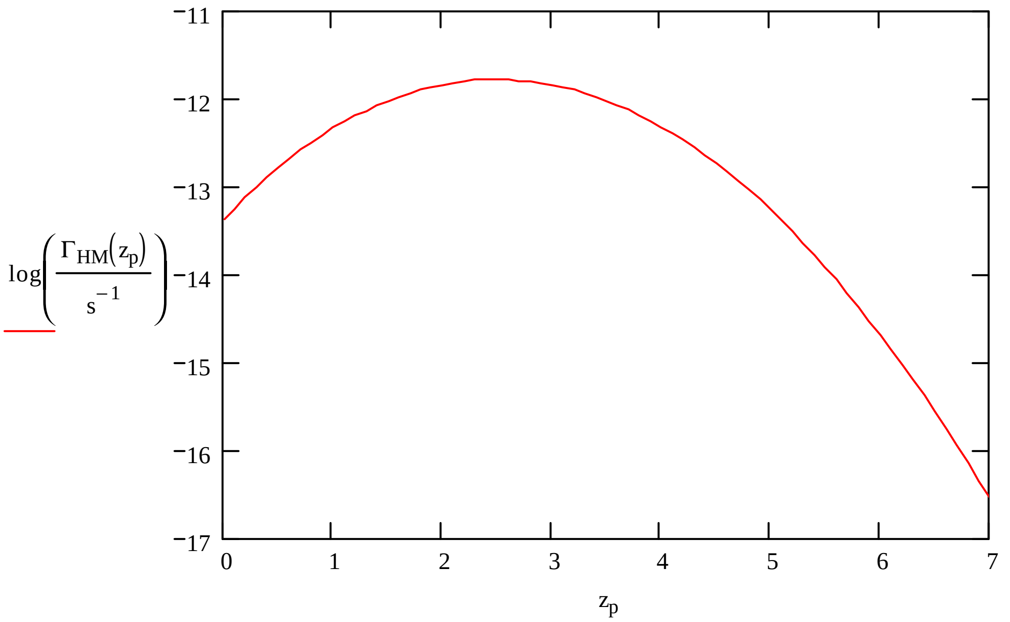
Functional Form for HI photoionisation rate as a function of redshift
Haardt and Madau 1996, ApJ, 461, 20 NB done for different cosmology - use here only as 'representative'

$A := 6.7 \cdot 10^{-13} \cdot s^{-1}$ $B := 0.73$ $z_c := 2.30$ $S := 1.90$

$$\Gamma_{HM}(z) := A \cdot (1+z)^B \cdot \exp\left[\frac{-(z-z_c)^2}{S}\right]$$

photoionisation rate per hydrogen atom

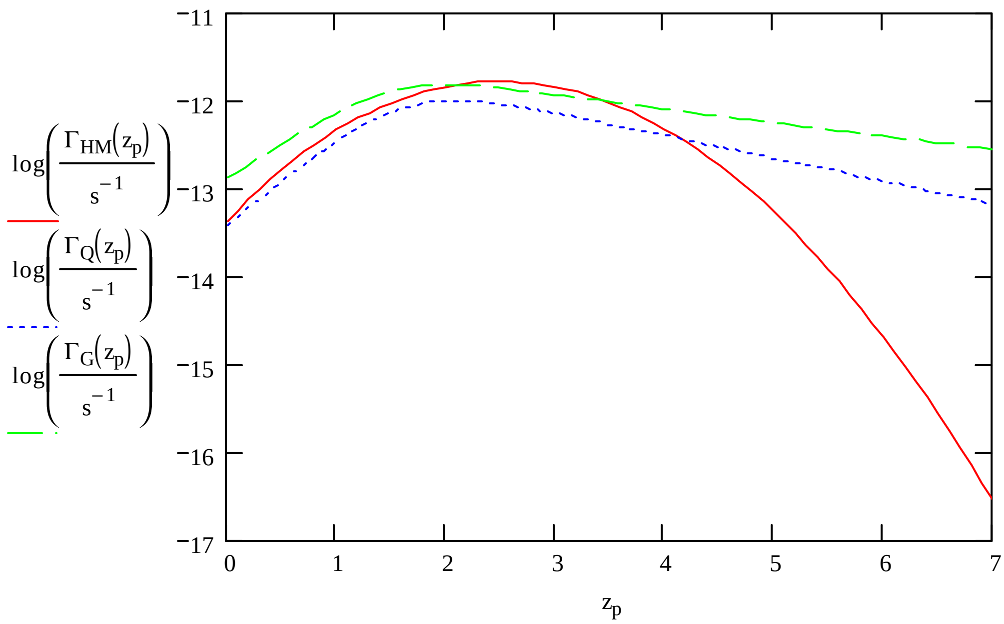
$z_p := 0, 0.1 .. 7$



Play around with new CUBA version of UV background (uses consensus cosmology)
Parameter values from Haardt web site for QSO only and QSO+Galaxy background.

$$CQ := \begin{pmatrix} 38.1263542 \\ 101.437042 \\ 2.272367 \\ 2.89428115 \\ 0.139637366 \end{pmatrix} \qquad CG := \begin{pmatrix} 61.4587822 \\ 45.5754623 \\ 2.39223647 \\ 2.64183688 \\ -0.64656541476 \end{pmatrix}$$

$$\Gamma_Q(z) := \frac{CQ_0 \cdot 10^{-13} \cdot s^{-1} \cdot e^{CQ_2 \cdot z}}{CQ_1 + e^{CQ_3 \cdot z}} (1+z)^{CQ_4} \qquad \Gamma_G(z) := \frac{CG_0 \cdot 10^{-13} \cdot s^{-1} \cdot e^{CG_2 \cdot z}}{CG_1 + e^{CG_3 \cdot z}} (1+z)^{CG_4}$$



$\Gamma := \Gamma_G$ choose which parameterisation to use

Hydrogen Photoionisation cross section (approximation from Gould and Weinberg 1996, ApJ, 468, 462)

$$\nu_0 := \frac{13.6056981 \cdot eV}{h_p} \qquad \alpha_T := 6.30 \cdot 10^{-18} \cdot cm^2$$

$$\sigma_{HI}(\nu) := \alpha_T \cdot \left(\frac{\nu}{\nu_0}\right)^{-2.75}$$

UV background spectrum

$\alpha := 1.73$ Bunker, Marleau and Graham 1998, AJ, 116, 2086

$$J_0(z) := \frac{\Gamma(z)}{4\cdot\pi\cdot sr\cdot\alpha_T}\cdot h_p\cdot(\alpha + 2.75)$$

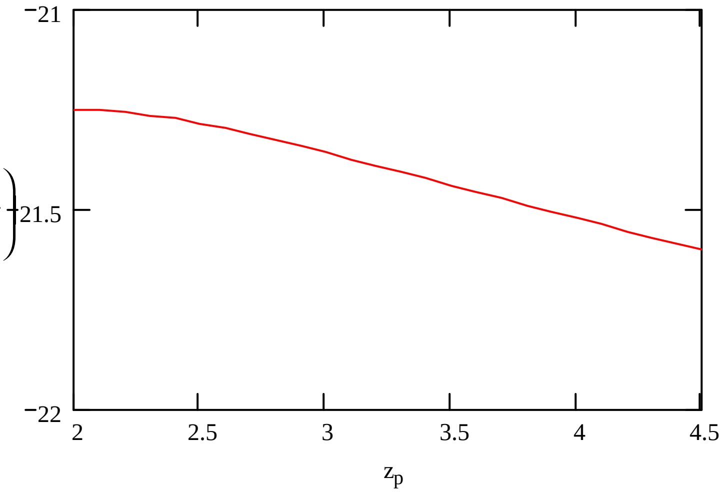
$$J_0(0) = 4.948\times 10^{-23}\, erg\cdot cm^{-2}\cdot s^{-1}\cdot Hz^{-1}\cdot sr^{-1}$$

$$J(\nu, z) := J_0(z)\cdot\left(\frac{\nu}{\nu_0}\right)^{-\alpha}$$

$$J_0(2.5) = 5.173\times 10^{-22}\, erg\cdot cm^{-2}\cdot s^{-1}\cdot Hz^{-1}\cdot sr^{-1}$$

UV background (matches abstract numbers for Haardt and Madau if use their Γ)

$$\log\left(\frac{J_0(z_p)}{erg\cdot cm^{-2}\cdot s^{-1}\cdot Hz^{-1}\cdot sr^{-1}}\right)$$



plot aimed at roughly matching Bunker et al fig 1. (NB possibly confused cosmology)

Lyman alpha luminosity of optically thick (i.e. in both Lyman alpha AND UV continuum) cloud.
Assume sphere with radius R (in cm) (Gould & Weinberg 1996, ApJ, 468, 462)

$\eta_{thick} := 0.62$ $z_{char} := 3$ $R_{char} := 50 \cdot kpc$ LLS size?

$$L_{thick}(R, z) := 4 \cdot \pi \cdot R^2 \cdot \eta_{thick} \cdot \left(\frac{J_0(z)}{h_p} \cdot \frac{2\pi \cdot sr}{\alpha} \right) \qquad L_{thick}(R_{char}, z_{char}) = 4.448 \times 10^{52} s^{-1}$$

Convert to line fluxes and surface brightnesses - opt thick (ignore edge dimming)

$$F_{thick}(R, z) := \frac{\frac{h_p \cdot c}{\lambda_{Ly\alpha}} \cdot L_{thick}(R, z)}{4 \cdot \pi \cdot d_L(z)^2} \qquad F_{thick}(R_{char}, z_{char}) = 9.398 \times 10^{-18} erg \cdot cm^{-2} \cdot s^{-1}$$

$$SB_{thick}(R, z) := \frac{F_{thick}(R, z)}{\pi \cdot \left(\frac{R}{d_A(z)} \right)^2} \qquad SB_{thick}(R_{char}, z_{char}) = 7.101 \times 10^{-20} erg \cdot cm^{-2} \cdot s^{-1} \cdot arcsec^{-2}$$

Lyman alpha luminosity of optically thin (i.e. in both Lyman alpha AND UV continuum) cloud.
Assume column density N (cm**-2), and cloud area πR**2 (Gould & Weinberg 1996, ApJ, 468, 462)

$\eta_{thin} := 0.42$ $N_{char} := 10^{16} \cdot cm^{-2}$

$$L_{thin}(N, R, z) := N \cdot \pi \cdot R^2 \cdot \eta_{thin} \cdot \Gamma(z) \qquad L_{thin}(N_{char}, R_{char}, z_{char}) = 3.665 \times 10^{50} s^{-1}$$

Convert to line fluxes and surface brightnesses - opt thin

$$F_{thin}(N, R, z) := \frac{\frac{h_p \cdot c}{\lambda_{Ly\alpha}} \cdot L_{thin}(N, R, z)}{4 \cdot \pi \cdot d_L(z)^2} \qquad F_{thin}(N_{char}, R_{char}, z_{char}) = 7.744 \times 10^{-20} erg \cdot cm^{-2} \cdot s^{-1}$$

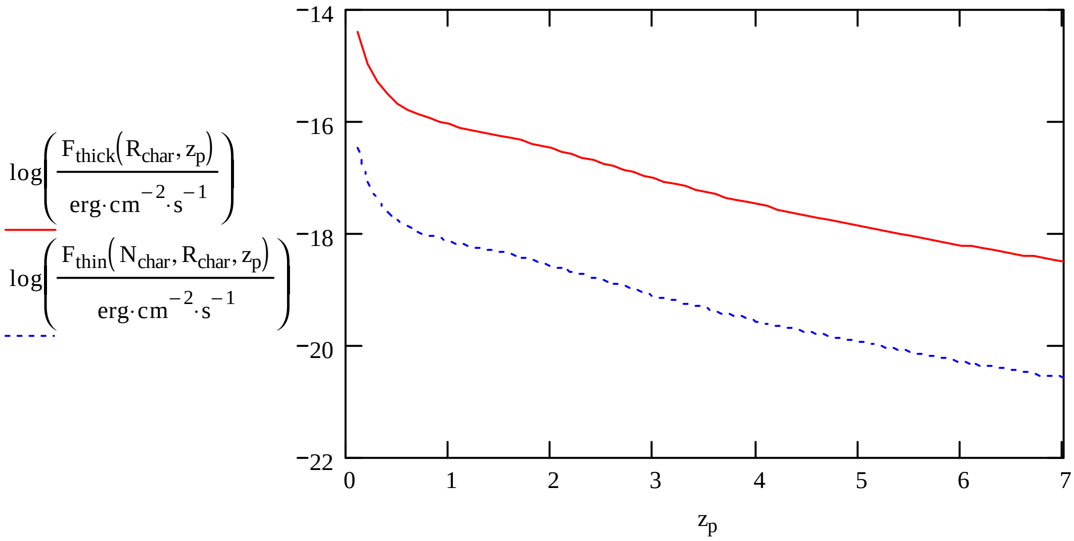
$$SB_{thin}(N, R, z) := \frac{F_{thin}(N, R, z)}{\pi \cdot \left(\frac{R}{d_A(z)} \right)^2} \qquad SB_{thin}(N_{char}, R_{char}, z_{char}) = 5.851 \times 10^{-22} erg \cdot cm^{-2} \cdot s^{-1} \cdot arcsec^{-2}$$

Formula from Bunker et al 1998 equation 2

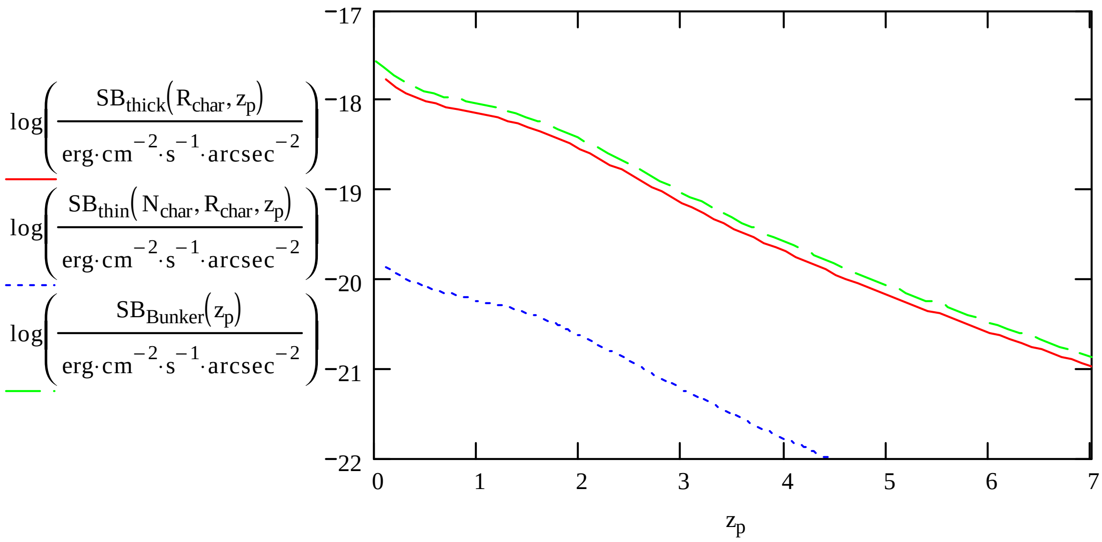
$$\eta_E := 0.5$$

$$SB_{Bunker}(z) := \frac{J_0(z)}{4.76} \cdot \left(\frac{\alpha - 1}{0.73}\right)^{-1} \cdot \left(\frac{\eta_E}{0.5}\right)^{-1} \cdot \left(\frac{1 + z}{4}\right)^{-4} \cdot 10^3 \cdot Hz \cdot sr \cdot arcsec^{-2}$$

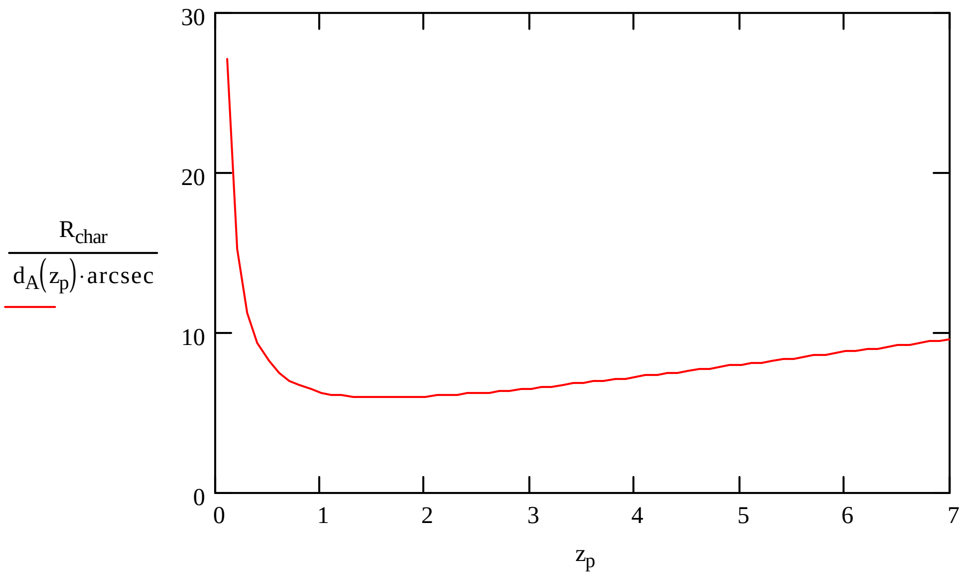
$$SB_{Bunker}(z_{char}) = 9.192 \times 10^{-20} \, erg \cdot cm^{-2} \cdot s^{-1} \cdot arcsec^{-2}$$



$\log\left(\frac{F_{thick}(R_{char}, z_p)}{erg\cdot cm^{-2}\cdot s^{-1}}\right)$
$\log\left(\frac{F_{thin}(N_{char}, R_{char}, z_p)}{erg\cdot cm^{-2}\cdot s^{-1}}\right)$
−14
−16
−18
−20
−22
0 1 2 3 4 5 6 7
z_p



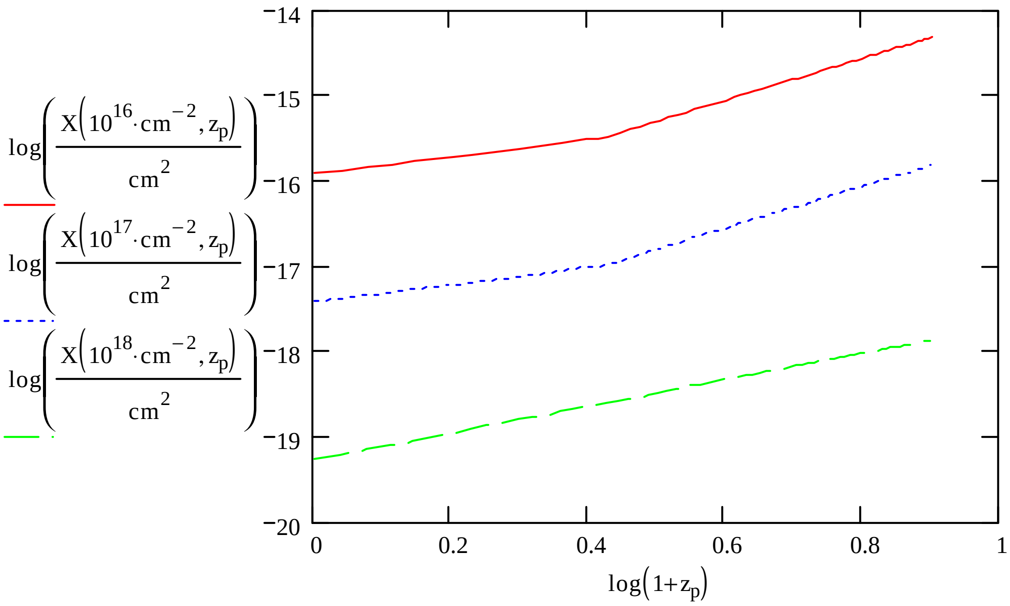
$\log\left(\frac{SB_{thick}(R_{char}, z_p)}{erg\cdot cm^{-2}\cdot s^{-1}\cdot arcsec^{-2}}\right)$
$\log\left(\frac{SB_{thin}(N_{char}, R_{char}, z_p)}{erg\cdot cm^{-2}\cdot s^{-1}\cdot arcsec^{-2}}\right)$
$\log\left(\frac{SB_{Bunker}(z_p)}{erg\cdot cm^{-2}\cdot s^{-1}\cdot arcsec^{-2}}\right)$
−17
−18
−19
−20
−21
−22
0 1 2 3 4 5 6 7
z_p



$\frac{R_{char}}{d_A(z_p)\cdot arcsec}$
30
20
10
0
0 1 2 3 4 5 6 7
z_p

Probability of intercepting a high column density region (Haardt and Madau 1996)

$$X(N_{HI}, z) := \begin{cases} 1.2 \cdot 10^{8} \cdot cm^{2} \cdot \left(\dfrac{N_{HI}}{cm^{-2}} \right)^{-1.5} \cdot (1+z)^{1.0} & \text{if } N_{HI} \ge 10^{13} \cdot cm^{-2} \wedge N_{HI} < 1.59 \cdot 10^{17} \cdot cm^{-2} \wedge z \le 1. \\ 2.9 \cdot 10^{7} \cdot cm^{2} \cdot \left(\dfrac{N_{HI}}{cm^{-2}} \right)^{-1.5} \cdot (1+z)^{2.46} & \text{if } N_{HI} \ge 10^{13} \cdot cm^{-2} \wedge N_{HI} < 1.59 \cdot 10^{17} \cdot cm^{-2} \wedge z > \\ 5.4 \cdot 10^{7} \cdot cm^{2} \cdot \left(\dfrac{N_{HI}}{cm^{-2}} \right)^{-1.5} \cdot (1+z)^{1.55} & \text{if } N_{HI} \ge 1.59 \cdot 10^{17} \cdot cm^{-2} \wedge N_{HI} \le 10^{20} \cdot cm^{-2} \\ 0 & \text{otherwise} \end{cases}$$

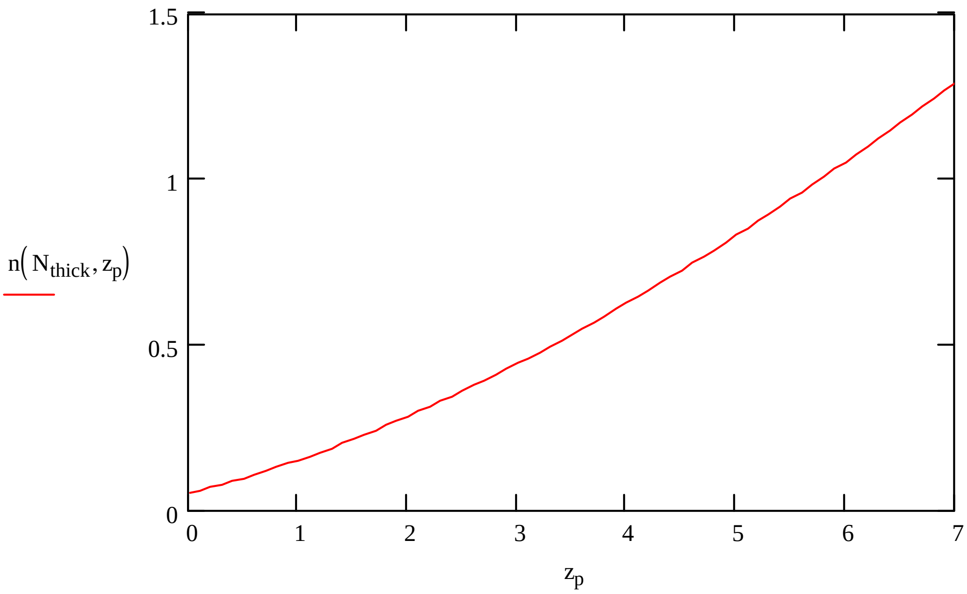


Integrate to get number of clouds above given column density per unit redshift at a given redshift

$$N_{thick} := 3\cdot 10^{18}\cdot cm^{-2}$$

$$n(N_{HI}, z) := \int_{N_{HI}}^{10^{20}\cdot cm^{-2}} X(x, z)\, dx$$

$$n(N_{thick}, z_{char}) = 0.442$$



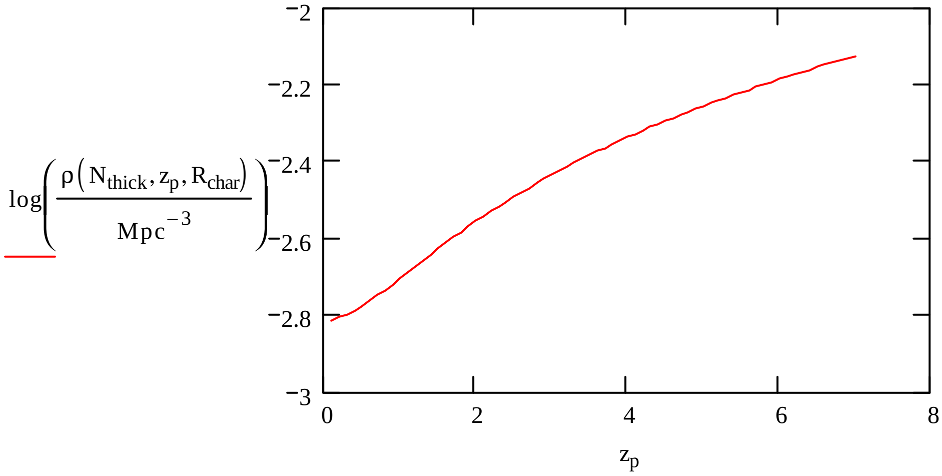
Now turn this into volume density assuming perfect spheres with radius R

$$F_{Kludge} := 1000$$

Kludged to avoiding thinking about GR and differential volumes

$$\rho(N_{HI}, z, R) := \frac{\frac{n(N_{HI}, z)}{F_{Kludge}}\cdot\left(arcmin\cdot d_A(z)\right)^2}{\pi\cdot R^2\cdot\frac{V\left(d_M\left(z + \frac{0.5}{F_{Kludge}}\right)\right) - V\left(d_M\left(z - \frac{0.5}{F_{Kludge}}\right)\right)}{\frac{4\cdot\pi}{arcmin^2}}}$$

$$\rho(N_{thick}, z_{char}, R_{char}) = 3.664\times 10^{-3}\, Mpc^{-3}$$



Plot of volume density of optically thick absorbers that would then satisfy the Lyman alpha flux and SB numbers for the optically thick case above. (Same mathCAD will also do optically thin)

> 0

0

.7

1.7